TEMPERATURE AND PRESSURE EQUILIBRIUM DISTRIBUTION IN A STEADY-STATE DISCHARGE WITH RADIATION

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Results of a numerical solution of the equations of energy and momentum balance are given for a steady-state discharge in a dense plasma.

One of the fundamental problems in the theory of quasistationary, heavy-current discharges is the determination of temperature and pressure equilibrium distributions, on which depends not only the energy balance but also the plasma stability. The balance equations were solved in [1] using various assumptions ($\varkappa = 0$, absence of radiation, and so on). Since the complete system of equations cannot be solved analytically, the balance equations were integrated numerically [2-4] for low densities, when radiation is unimportant. It is of interest to consider an equilibrium discharge in a dense plasma, where radiation becomes quite important. As shown below, quasiperiodic solutions may exist in this case.

Consider an equilibrium, cylindrical plasma column without a longitudinal magnetic field, where the plasma pressure is confined mainly by the intrinsic magnetic field of the current flowing along the z axis (part of the plasma pressure may be imparted directly to the container). Joule heat is removed by thermal conductivity and radiation, which is assumed to fill the whole volume, as is valid for a wide range of temperatures and densities. Assuming $T_e = T_i$, and neglecting viscosity, we obtain

$$\begin{gathered} dp/dr = c^{-1}\,\mathbf{fH}, \qquad \mathrm{rot}\,\mathbf{H} = 4\pi c^{-1}\mathbf{f} \\ \mathrm{div}\,(\varkappa \nabla T) = -\mathbf{Ef} + W_r(T, n), \qquad \sigma \mathbf{E} = \mathbf{j} \\ W_r(T, n) = \alpha n^2 T^{-1/2} + \beta n^2 T^{1/2} \end{gathered} \tag{1}$$

Here σ is the conductivity, **H** is the magnetic field, n is the density, T is the temperature, p is the pressure, **j** is the current density, **E** is the electric field, $\varkappa$ is the coefficient of thermal conductivity, and α and β are the Bremsstrahlung and recombination radiation coefficients [5]. The effects considered are connected with radiation; i.e., they occur in the well-known dense plasma. We can therefore consider, without loss of generality, a fully ionized, unmagnetized plasma, $\sigma = \sigma_0 T^{3/2}\lambda^{-1}$, $\varkappa = \varkappa_0 T^{5/2}\lambda^{-1}$, σ_0, $\varkappa_0 = \mathrm{const}$.

Assuming the Coulomb logarithm λ to be constant, we rewrite the system of equations (1) in dimensionless form:

$$\begin{gathered} \frac{1}{x}\frac{d}{dx}\left(\frac{x}{\theta^{3/2}}\frac{d\rho}{dx}\right) = -\theta^{3/2} \\ \frac{1}{x}\frac{d}{dx}\left(x\theta^{5/2}\frac{d\theta}{dx}\right) = -q_J\theta^{3/2} + q_r f(\theta)\rho^2 \\ f(\theta) = \theta^{-5/2}(1 + \beta_1\theta), \qquad \beta_1 = \beta T_0/\alpha \\ \theta = \frac{T}{T_0}, \quad \rho = \frac{p}{p_0}, \quad x = \frac{r}{R}, \quad R^2 = \frac{p_0}{4\pi j_0^2} \\ q_J = \frac{j_0^2 R^2 \lambda^2}{\sigma_0 T_0^{3/2}\varkappa_0 T_0^{7/2}}, \qquad q_r = \frac{\alpha n_0^2 R^2 \lambda}{T_0^{1/2}\varkappa_0 T_0^{7/2}} \end{gathered} \tag{2}$$

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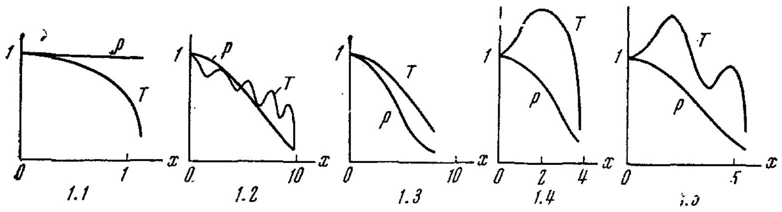

Fig. 1

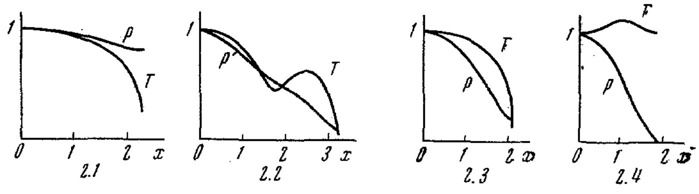

Fig. 2

with boundary conditions

$$\rho = 1, \quad \theta = 1 \quad \text{at} \quad x = 0$$

(The index 0 is related to the value at $r = 0$.)

Consider first some possible types of solutions of the system (2).

A. Small radiation, $q_r \ll q_J$. Integrating (2) once, we have

$$\rho\ (x) - 1 = {}^1/_5\, (\theta^5\,(x) - 1)\ /\ q_J.$$

At $5q_J > 1$ the magnetic pressure is low and the solution corresponds to a steady-state arc. At $5q_J < 1$ the solution corresponds to a self-constricted discharge with thermal losses through the surrounding channel currents to the gas sheath. A special case is $5q_J = 1$, when the heat flow to the vessel is minimal.

B. Low magnetic pressure, but the radiation power is comparable to the Joule heating power, $q_J \sim q_r \gg 1$.

In this case the solution corresponds to a high-temperature arc. Reducing the energy balance equation to an equation of motion of a particle in potential field

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{d\varphi}{dx}\right) = -\frac{dV(\varphi)}{d\varphi}$$

it is not hard to verify [6] the possibility of an oscillatory system.

In the general case, Eqs. (1) and (2) can be solved only numerically. Figures 1 and 2 show the temperature and pressure equilibrium distributions corresponding to the values $q_r = 0$, 0.1, 0.5, 1.0, and 2.0 at $q_J = 0.1$ (Fig. 1) and $q_r = 0$, 0.5, 1.0, and 2.0 at $q_J = 1$ (Fig. 2). The figures show that steady-state temperature and pressure equilibrium distributions satisfying the criterion of plasma stability do not exist for all possible discharge parameters. This is true both for fast vibrations (of a period less than the skipping time) and for slow ones [7, 8]. Solutions are possible for $q_J \ll 1$ (Figs. 1.1 and 2.1), when the magnetic pressure is small compared with the gas kinetic one (an electric arc). Solutions corresponding to self-constricted discharge either isolated from the walls (Fig. 2.4) or part of its plasma pressure is removed by the walls (Figs. 1.3 and 2.3) exist for the given values of temperature and density only along the discharge axis at definite values of the electric field. In the remaining cases the solution oscillates (Figs. 1.2 and 2.2), the plasma column being unstable with such temperature and pressure distributions.

The solutions found only illustrate various types of temperature and pressure equilibrium distributions for high-density discharges. For a correct solution of the problem of discharge equilibrium, it would be necessary to consider the magnetic field dependence of the thermal conductivity coefficient [3] and the dissociation and ionization energy transfer [4]. This limitation, however, is not important, as the qualitative nature of the solution is independent of the form of $\varkappa$ (T, H) and the latter effect is significant only at temperatures below the ionization temperature.

LITERATURE CITED

1. R. S. Piz, "Equilibrium characteristics of a constricted gas discharge cooled by Bremsstrahlung," in: Controlled Thermonuclear Reactions [in Russian], Atomizdat, Moscow (1960).
2. H. Alfvén and E. Smars, "Gas insulation of a hot plasma," Nature, 188, 801 (1960).
3. S. G. Alikhanov and S. P. Ivaniya, "Temperature and density distribution in a high-temperature steady-state arc," Zh. Tekh. Fiz., 35, 557 (1965).
4. G. K. Verboom, "The energy balance of an arc discharge in hydrogen gas," Plasma Physics, 11, 903 (1969).
5. S. A. Kaplan and S. B. Pikel'ner, Interstellar Media [in Russian], Fizmatgiz, Moscow (1963).
6. S. G. Alikhanov, I. K. Konkashbaev, and P. Z. Chebotaev, "The energy balance in a dense fusion plasma contained by walls," Nuclear Fusion, 10, 13 (1970).
7. B. B. Kadomtsev, "Hydromagnetic plasma stability," in: Reviews of Plasma Physics, Vol. 2, Consultants Bureau (1966).
8. I. K. Konkashbaev, "Current instability of a finite conductivity plasma," Zh. Prikl. Mekhan. i Tekhn. Fiz., Vol. 11, No. 2 (1970).